IGA/AMSI Workshop on Geometric Quantisation

Titles and Abstracts

27-31 July 2015
Ingkarni Wardli 7.15

Monday, 27 July

- Prof. Michèle Vergne (Institut de Mathématiques de Jussieu) 9:30-10:30
  - Title: A Formula for the Multiplicities of the Index of Transversally Elliptic Operators
  - Abstract: Let M be a manifold, provided with an action of a torus T. Let D be a T-invariant pseudo-differential operator on M, which is T-transversally elliptic. I will give a formula for the equivariant index of D, using the formal T-equivariant $\hat{A}$-class. Then, using the notion of infinitesimal index (as defined in De Concini-Procesi-Vergne), this formula constructs a piecewise locally polynomial function m on Lie(T)*, depending only of the principal symbol of D such that index(D)$(g) = \sum_{\lambda \in \hat{T}} m(\lambda) g^{\lambda}$. Geometric interpretation of $m(\lambda)$ can be given for Dirac operators twisted by line bundles.
- Prof. Maxim Braverman (Northeastern University) 11:00-12:00
  - Title: Index Theorem for Transversally Elliptic with Parameter Operators on Complete Manifolds
  - Abstract: We define an analytical index of a class of differential operators on a complete G-manifold M, called operators transversally elliptic with parameter. The symbol with parameter of such an operator defines an element of transversally elliptic K-theory of M. We show that the topological index of this symbol is equal to the analytical index of the operator. We also show that this index is invariant under a class of non-compact cobordisms. As an application of this result we obtain a gluing formula and an Atiyah-Bott type fixed point formula for our index. If M is a Hamiltonian G-manifold and the set of zeros of the vector field generated by the moment map is compact, then the Tian-Zhang deformation of the $Spin^c$ Dirac operator on M is an operator transversally elliptic with parameter. In this case our index gives a geometric quantization of M.
- Prof. Siye Wu (National Tsing Hua University) 2:00-3:00
  - Title: Non-orientable Surfaces and Electric-magnetic Duality
  - Abstract: In this talk, I explain the role of non-orientable surfaces in twisted N=4 supersymmetric gauge theory in 4 dimensions whose compactification along orientable surfaces yields mirror symmetry and geometric Langlands program as studied by Kapustin and Witten. I relate the discrete electric and magnetic fluxes of ?t Hooft in 4 dimensions to the topology of moduli spaces from 2 dimensions.

- Dr. Rung-Tzung Huang (National Central University) 3:30-4:30
  - Title: The Comparison of Two Constructions of the Refined Analytic Torsion on Compact Manifolds with Boundary
  - Abstract: The refined analytic torsion was introduced by M. Braverman and T. Kappeler as a refinement of analytic torsion on odd dimensional closed manifolds. It has been extended to the case on odd dimensional compact manifolds with boundary by B. Vertman and by Yoonweon Lee and myself in different ways. In this talk I will talk about the comparison of these two constructions of the refined analytic torsion on compact manifolds with boundary. This is a joint work with Yoonweon Lee.

Tuesday, 28 July

- Prof. Weiping Zhang (Chern Institute of Mathematics) 9:30-10:30
  - Title: Various Aspects of Analytic Localization
  - Abstract: I will try to discuss the basic strategy of the Bismut-Lebeau analytic localization technique and describe various applications in geometric quantization as well as some other aspects.
- Dr. Jennifer Vaughan (University of Toronto) 11:00-12:00
  - Title: A Dynamically Invariant Quantized Energy Condition for Metaplectic-c Quantizable Systems
  - Abstract: Robinson and Rawnsley developed an alternative to the classical Kostant-Souriau quantization procedure with half-form correction in which the prequantization circle bundle and metaplectic structure are replaced by a single object called a metaplectic-c prequantization. I will begin by describing the structure of a metaplectic-c prequantization. Then, given a metaplectic-c quantizable symplectic manifold and a Hamiltonian energy function, I will propose a condition under which a regular value of the Hamiltonian can be considered a quantized energy level of the system. This condition depends only on the geometry of the level set, and not on the dynamics of a particular Hamiltonian. When applied to the harmonic oscillator and the hydrogen atom, it correctly reproduces the standard quantum mechanical energy levels.
- Prof. Paul-Émile Paradan (Université Montpellier 2) 2:00-3:00
  - Title: $[Q, R] = 0$ for spinc Dirac operators
  - Abstract: In the preprint arXiv:1411.7772 done with Michle Vergne, we have given a geometric description of the multiplicities of the equivariant index of $spin^c$ Dirac operators, in the spirit of the Guillemin-Sternberg phenomenon $[Q, R] = 0$. In this talk we will explain the main ingredients of the proof.
- Dr. Fei Han (National University of Singapore) 3:30-4:30
  - Title: Witten Rigidity on Odd Dimensional Manifolds
  - Abstract: An elliptic operator on a manifold is called rigid if its equivariant index is constant or vanishing when the manifold admits a nontrivial Lie group action. Classical operators like the signature operators, the Dirac operators and etc. are rigid. The Witten rigidity conjecture predicts the rigidity of many important twisted Dirac operators on even dimensional spin manifolds and was proved by Bott, Taubes and Liu. In this talk, I will describe Witten type rigidity theorems on odd dimensional spin manifolds. This represents our joint work with Jianqing Yu.

Wednesday, 29 July

- Prof. Eckhard Meinrenken (University of Toronto) 9:30-10:30
  - Title: Dirac Geometry of the Holonomy Fibration.
  - Abstract: Let A_{S^1} be the space of G-connections over a circle, equipped with the gauge action of the loop group LG. The subgroup L_0G of based loops acts freely, defining a principal bundle $\mathrm{Hol} : A_{S^1} \to G$ with quotient map the holonomy. If the Lie algebra of G has an invariant metric, then $\mathcal{A}$ carries an invariant 'Lie-Poisson structure', with symplectic leaves the coadjoint loop group orbits. Taking a quotient by L_0G, one might expect to get a Poisson structure on G. I will explain why this does not work, due to infinite-dimensional problems, and how the correct quotient procedure instead gives a Dirac structure on G. (Joint work with Alejandro Cabrera and Marco Gualtieri.)
- Dr. Yiannis Loizides (University of Toronto) 11:00-12:00
  - Title: Norm-square Localization for Hamiltonian LG-spaces
  - Abstract: Let $\psi : \mathcal{M} \to L\mathfrak{g}^*$ be a proper Hamiltonian LG-space. Dividing out by the free action of the based loop group L_0G, yields a finite dimensional quasi-Hamiltonian space $\Phi : M \to G$. It is possible to define twisted Duistermaat-Heckman (DH) measures for (M, Φ), which are distributions on G encoding cohomological pairings on reduced spaces. Similar to the Hamiltonian case, these can be computed using abelian localization. We will discuss an analogue of norm-square localization—as initiated by Witten and studied in depth by Paradan—for twisted DH-measures of quasi-Hamiltonian spaces, where the terms of the formula are indexed by the components of the critical set of $||\psi||^2$.

Thursday, 30 July

- Prof. Nigel Higson (Pennsylvania State University) 9:30-10:30
  - Title: Geometric Quantization and Categories of Smooth Representations
  - Abstract: My talk will be about an effort to understand some of Harish-Chandra' s ideas in representation theory through geometry and geometric analysis. I will focus on parabolic induction, which is the main process used to create representations of a reductive group like $\mathrm{GL}(n, \mathbb{R})$ from (presumably simpler) representations of subgroups. In its first instances parabolic induction is a prime example of geometric quantization in representation theory. I will also discuss parabolic restriction, which is the main process used to investigate general representations in terms of (presumably simpler) representations of subgroups. Bernstein showed that, within the context of p-adic groups, parabolic induction and restriction are related by two crucial reciprocity laws. Together they help establish the broad structure of representation theory. The more difficult of the two laws fails for real groups, but I will explain how it can be re-established within Harish-Chandra' s context of tempered representations, using the wave equation and other geometric and analytic ideas. This is joint work with Tyrone Crisp (Copenhagen) and Robert Yuncken (Clermont-Ferrand).

- Dr. Yanli Song (University of Toronto) 11:00-12:00
  - Title: Equivariant Indices of Spin-c Dirac Operators for Proper Moment Maps.
  - Abstract: Given a compact, connected Lie group acting on a possibly non-compact manifold, we can associate it with an equivariant map from the manifold to the Lie algebra, which generalizes the moment map introduced in the symplectic case. Under the assumption that the moment map is proper, we will explain how to define an equivariant index of Spin-c Dirac operators on the manifold and decompose the index into irreducible representations according to a quantization commutes with reduction principle. This joint work with Peter Hochs (University of Adelaide).
- Prof. Anthony Henderson (University of Sydney) 2:00-3:00
  - Title: Towards a Theory of Modular Character Sheaves
  - Abstract: Lusztig's theory of character sheaves on a connected reductive algebraic group G is a geometric version of the ordinary (characteristic-0) representation theory of the corresponding finite groups $G(\mathbb{F}_q)$. In particular, it incorporates geometric versions of parabolic induction and restriction, whose definition involves derived push-forward functors. The analogue of the fact that characteristic-0 representations of finite groups are semisimple is provided by the Decomposition Theorem for a derived push-forward. If one wants a similar geometric version of the modular (characteristic-ℓ) representation theory of $G(\mathbb{F}_q)$, one should consider modular sheaves on G. In the modular setting, everything is harder: semisimplicity fails and so does the Decomposition Theorem. As a first step towards modular character sheaves, we have studied the geometric induction and restriction functors for sheaves on the nilpotent cone of G, proving analogues of Lusztig's results. This is a joint project with Pramod Achar, Daniel Juteau and Simon Riche.
- Dr. Romero Solha (Universidade Federal de Minas Gerais) 3:30-4:30
  - Title: Real Geometric Quantisation
  - Abstract: I intend to show some known results about geometric quantisation with real polarisations provided by integrable hamiltonian systems, and how to exploit the existence of circle actions to reprove those results and to compute the relevant cohomology groups for new cases (e.g. almost toric symplectic manifolds).